Bachelor of Science (B.Sc.) Semester—III (C.B.S.) Examination

MATHEMATICS (Differential Equations & Group Homomorphism)

Paper—II

Time : Three Hours] [Maximum Marks : 60

Note :— (1) Solve all the **FIVE** questions.

(2) All questions carry equal marks.

(3) Question Nos. **1** to **4** have an alternative. Solve each question in full or its alternative in full.

UNIT—I

1. (A) Prove the recurrence relation :

$$2nJ_n(x) = x\,[J_{n-1}(x) + J_{n+1}(x)].$$ 6

(B) Prove that :

$$\frac{1}{2} x\, J_n = (n+1)\, J_{n+1} - (n+3)\, J_{n+3} + (n+5)\, J_{n+5} - ....$$ 6

OR

(C) Prove that $P_n(x) = \dfrac{1}{2^n . n!} \dfrac{d^n}{dx^n} (x^2 - 1)^n$. 6

(D) By using Rodrigue's formula, show that :

$$\int_{-1}^{1} P_n(x)\, dx = 0 \quad ; \text{if } n \geq 1.$$ 6

UNIT—II

2. (A) If f(t), f′(t) are continuous for t > 0 and L[f(t)] = F(s), then prove that L[f′(t)] = s F(s) – f(0). Hence find L[cos at] by assuming L[sin at] = $\dfrac{a}{s^2 + a^2}$. 6

(B) Show that :

(i) $L^{-1}\dfrac{1}{(s-2)^2} = t\,e^{2t}$

(ii) $L^{-1}\log\left(1+\dfrac{1}{s^2}\right) = \dfrac{2(1-\cos t)}{t}$. 6

OR

(C) Let f(t) be a periodic function with period p, where p > 0, then prove that :

$$L[f(t)] = \frac{1}{1-e^{-sp}}\int_o^p e^{-su}\, f(u)\, du.$$ 6

(D) Evaluate $L^{-1}\dfrac{s}{(s^2+1)(s^2-4)}$ by using convolution theorem. 6

UNIT—III

3. (A) Solve $y'' + y = \cos 2t$, given that $y(0) = 1$, $y'(0) = -2$. 6

(B) Solve $x' + 5x + 2y = t$, $y' + 2x + y = 0$, where $x(0) = 0$, $y(0) = 0$ and '′' represents derivative w.r.t. t. 6

OR

(C) Let u(x, t) be a function defined for t > 0 and $x \in [a, b]$. Show that :

(i) $L\left(\dfrac{\partial u}{\partial t}\right) = s\,U - u(x, 0)$, where $U = u(x, s) = L[u(x, t)]$.

(ii) $L\left[\dfrac{\partial^2 u}{\partial t^2}\right] = s^2 U - s\, u(x, 0) - u_t(x, 0)$, where $u_t(x, 0) = \dfrac{\partial u}{\partial t}$ at t = 0. 6

(D) Find the Fourier transform of $f(x) = \begin{cases} 1, & \text{for } |x| < 1 \\ 0, & \text{for } |x| > 1 \end{cases}$. Hence, evaluate $\displaystyle\int_0^\infty \frac{\sin x}{x}\,dx$. 6

UNIT—IV

4. (A) Define normal subgroup of a group. Prove that the intersection of two normal subgroups of a group is a normal subgroup. 6

(B) Define a cyclic group. Show that every quotient group of a cyclic group is cyclic but not conversely. 6

OR

(C) Let K be the kernel of homomorphism f of a group G into a group G'. Prove that K is a normal subgroup of G. 6

(D) Let G be the additive group of real numbers and G' be the multiplicative group of all positive real numbers. Then show that a mapping $f : G \rightarrow G'$ defined by $f(x) = e^x, \forall x \in G$ is an isomorphism of G onto G'. Also find kernel of f. 6

Question—5

5. (A) Show that $J'_0 = -J_1$. 1½

(B) Prove that $\frac{1}{3}P_0(x) + \frac{2}{3}P_2(x) = x^2$. 1½

(C) Find $L(3\,e^{2t} - 5)^2$.

(D) Evaluate $L[t \,.\, \cos 2t]$. 1½

(E) Show that $L\left[\frac{\partial^2 u}{\partial x^2}\right] = \frac{d^2 u}{dx^2}$, where $U = L[u(x, t)]$. 1½

(F) Find the Fourier cosine transform of the function $f(x) = e^{-ax}$. 1½

(G) Let G and G′ be two groups and $f : G \rightarrow G'$ be a homomorphism. Then prove that :

$f(x^{-1}) = [f(x)]^{-1}, \forall x \in G$. 1½

(H) Show that a multiplicative group $G = \{1, \omega, \omega^2\}$, with $\omega^3 = 1$ is cyclic. How many are the generators ? 1½